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# Absolute Instability in a Supersonic Shear Layer and Mixing Control

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**The effects of the initial wake deficit on the instability of a confined supersonic shear layer are investigated. The wake deficit is generated by a thick splitter plate. If the wake deficit is sufficiently large, the flow changes from being convectively to absolutely unstable. Physically, the absolute instability acts like an oscillator and the flow behaves in a similar manner as under an externally driven excitation, except that the flow is self-excited. It is argued that, if the frequency of the absolute instability coincides with that of an unstable convective instability further downstream, this instability may be excited, which could lead to mixing enhancement. To simulate the effect of the wake, a wake deficit is superimposed onto a monotonic mean velocity profile. The mean density is calculated through the Crocco–Busemann relation. Numerical methods that use the Briggs–Bers pinching criterion are implemented. The numerical results indicate that increasing the velocity difference between the two streams and cooling the low-speed stream facilitates the transition of the supersonic shear layer from convective to absolute instability.**

## Introduction

THE development of a high-speed transportation vehicle remains a national research interest. One of the technical challenges in the development of such a vehicle is its propulsion system. To optimize the efficiency of the diffuser and to avoid dissociation/reassociation after fuel combustion, the combustor inlet Mach number for a hypersonic propulsion system is generally supersonic.[1] At such a high speed, the ejected fuel momentum accounts for a large fraction of the engine thrust. Hence, parallel fuel ejection is preferred.[2] When the fuel is injected in this manner, a parallel compressible shear layer is formed. However, such tangential injection shortens the residence time in the combustor, and the high-speed parallel flow mixes and spreads very slowly compared to mixing in incompressible flow for the same velocity and density ratios. This situation becomes more severe with increasing convective Mach numbers.[3] Thus, the successful use of a supersonic combustor as a propulsion system in hypersonic flight vehicles depends critically on techniques to enhance the level of mixing. A review of supersonic mixing enhancement techniques can be found in Gutmark et al.[4] and Bushnell.[5] In these papers, most mixing enhancement techniques described either suffer severe performance losses or would be difficult to implement reliably at full scale in harsh environments. Thus, a mixing enhancement scheme should have the following features to be practical. First, it must provide sufficient mixing with a minimum performance penalty. Second, the implementation of any mixing enhancement method must be practical in an engineering sense.

A mixing enhancement scheme that meets these two criteria is discussed in this paper. This scheme uses the concept of a local absolute instability. An absolute instability in the initial shear layer behaves in a similar manner to an external excitation and provides continuous temporal excitation to the downstream flowfield. If the frequency of the absolute instability coincides with the most unstable downstream convective instability, it would be excited and mixing enhancement could occur. However, note that the experiments of Strykowski and Niccum[6] and Oster and Wygnanski[7] show that the presence of the local absolute instability or external forcing may actually suppress the mixing. Thus, the present suggestion that mixing enhancement might occur needs to be demonstrated by experiment.

Linear stability calculations by Pavithran and Redekopp[8] and Peroomian and Kelly[9] both show that reversed flow is required to change the flow instability from convective to absolute for confined shear layers. Thus, an immediate concern is how to generate the reversed flow. If a vacuum pump is needed to generate the reversed flow, such as that used by Strykowski and Niccum[6] in a related application, the overall engine efficiency would decrease because of the extra power drain and weight of the pump. This paper resolves this concern by proposing that the use of a thick splitter plate in a confined supersonic shear layer could generate sufficient reversed flow. The flow behind a blunt splitter plate generally possesses a wake component caused by viscous separation effects. If the splitter plate is thick enough, it can generate a strong reversed flow,[10] which will be shown to induce an absolute instability without the need for suction.

This paper is organized in the following manner. First, the mathematical formulation is provided. Then the numerical methods are described. Because the manipulation of the absolute instability frequency is of primary interest, numerical calculations based on different operating conditions are performed. General guidelines are established for the manipulation of the absolute frequency and some conclusions are drawn.

## Mathematical Formulation and Numerical Methods

The equation governing waves in a confined, two-dimensional shear layer, after linearization and combination of variables, is the compressible Rayleigh equation given by

$$\frac{d^2\hat{p}}{dy^2} + \left[\frac{2K}{(\omega - K\bar{u})}\frac{d\bar{u}}{dy} - \frac{1}{d\bar{\rho}}\frac{d\bar{\rho}}{dy}\right]\frac{d\hat{p}}{dy} + [\bar{\rho}M_1^2(\omega - K\bar{u})^2 - K^2]\hat{p} = 0 \qquad (1)$$

where $\hat{p}$, $\bar{u}$, $\bar{\rho}$, and $M_1$ are the fluctuation pressure, mean velocity, mean density, and high-speed stream Mach number, respectively. $K$, $\omega$, and $y$ are the complex wave number, radian frequency, and transverse coordinate, respectively. All of the

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flow variables in Eq. (1) are nondimensionalized by the corresponding high-speed stream quantities. The channel height $H$ is taken to be the length scale. The boundary conditions at the walls are

$$\frac{d\hat{p}}{dy} = 0 \quad \text{at} \quad y = \pm\frac{1}{2} \tag{2}$$

To simulate the wake behind a thick splitter plate, a wake deficit is superimposed onto a monotonic velocity profile so that the streamwise velocity profile takes the following form:

$$\bar{u}(y) = \frac{1}{2}[1 + \lambda_u + (1 - \lambda_u)\tanh(gy)] - \Lambda\exp[-f(gy)^2] \tag{3}$$

where $\Lambda$ is a constant that sets the magnitude of the velocity deficit behind the splitter plate. The variable $f$ controls the width of the wake, and $g$ modifies the shear-layer vorticity thickness. $\lambda_u$ is the velocity ratio between the low- and high-speed streams. In the subsequent numerical simulations, $g$ and $f$ are fixed at 40 and 0.4, respectively. The mean density profile is calculated through the use of the Crocco–Busemann relation. Though the preceding equations are similar to those used by Zhuang and Dimotakis[11] and to those used in many linear stability analyses, the purpose and numerical approach here are quite different. In this paper, we are interested in finding the wake deficit $\Lambda_{req}$ that just alters the flow instability from convective to absolute. Second, different flow parameters are used as control variables to determine the magnitude of the wake deficit required to induce an absolute instability. Finally, these control parameters are also used to modify the absolute instability frequency to match a desired self-excitation frequency.

Two approaches may be used in the solution of Eq. (1). One is a spatial analysis that specifies a real frequency $\omega$ and searches for the possible complex wave numbers $K$ as eigenvalues. The other is a temporal analysis that specifies a real wave number and looks for possible complex frequencies as eigenvalues. For a convectively unstable flow, a spatial analysis is more appropriate. However, for an absolutely unstable flow, a temporal analysis more closely models the physics. Because the transition of the type of flow instability is of interest here, a spatial–temporal analysis is used. In a spatial–temporal analysis, both the wave number and frequency are complex numbers. It is necessary to use this approach to investigate the transition between convectively and absolutely unstable flows, as noted by Huerre and Monkewitz.[12] Another point worth noting is that for cases where the branch cut from the singularity at $\omega - K\bar{u}$ crosses the real $y$ axis, the integration must be deformed into the complex $y$ plane below the critical point, but above any singularities of the hyperbolic tangent function.

As noted in Refs. 13–15, the saddle point in the frequency/wave number relationship provides crucial information about the nature of the instability of the wake. To facilitate an explanation of the numerical procedure and further discussion, an operating condition of $M_1 = 3.0$, $M_2 = 1.2$, $T_{t1} = T_{t2} = 293$ K, $\gamma_1 = \gamma_2 = 1.4$ is chosen. First, the value for the wake deficit in the velocity profile is specified. Then both responses $K^+(\omega)$ and $K^-(\omega)$ corresponding to $x > 0$ and $x < 0$ for different $\omega$ are determined until a saddle pattern is observed. $K^+$ and $K^-$ are the complex mappings with imaginary parts greater or less than zero, respectively, along a particular Laplace contour. For a convectively unstable flow, the growth rate (minus the imaginary part of the complex wave number $-K_i$) increases to a maximum, then decreases through zero as frequency is increased. A typical convective instability is the Kelvin–Helmholtz instability in the shear layer. However, for an absolutely unstable flow, the growth rates only increase with frequency.

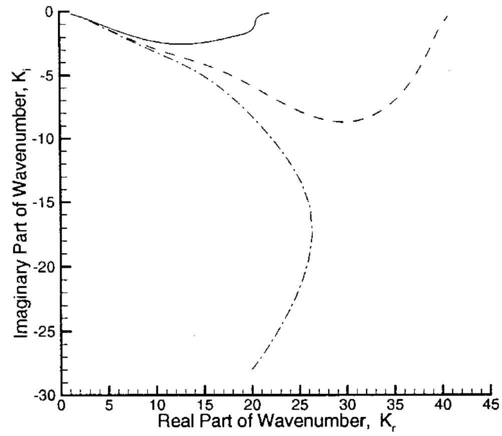


**Fig. 1 Loci of $K^+$ before and after the onset of an absolute instability. $M_1 = 3.0$, $M_2 = 1.2$ ——, L = 0.67; - - -, L = 0.77; – - - -, L = 0.80.**

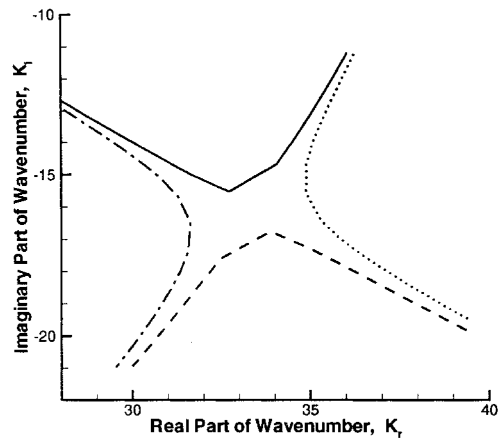


**Fig. 2 Loci of $K^+$ and $K^-$ before and after the onset of an absolute instability. $M_1 = 3.0$, $M_2 = 1.2$. ——, L = 0.7909, $K^+$; - - -, L = 0.7909, $K^-$; – - - -, L = 0.7915, $K^+$; ······, L = 0.7915, $K^-$.**

This can be appreciated by plotting the variation of the real and imaginary parts of the complex wave number with frequency (Fig. 1). The slight kink in the curve for $\Lambda = 0.67$ is typical of supersonic shear-layer growth rate curves near the neutral point[16] and is not indicative of an incipient absolute instability.

The qualitative characteristics of the $K$ map are shown in Fig. 2. Near the center of these four curves, there is a saddle point. To refine the saddle point, a second numerical technique has been developed. This method utilizes the fact that, at the saddle point, the frequency $\omega_0$ is, in general, a branch point of order 2; that is, locally, it is a square root branch point. For two initial frequencies, the corresponding eigenvalues $K^+$ and $K^-$ are determined. Then the branch point is interpolated by fitting these data to an expression of the form

$$K^{\pm} - K_0 = \pm c_1(\omega - \omega_0)^{1/2} + c_2(\omega - \omega_0) \tag{4}$$

where $K_0$ and $\omega_0$ are the wave number and frequency at the saddle point, and $c_1$ and $c_2$ are constants. For two initial frequencies, four equations are obtained. This allows the four unknowns ($\omega_0$, $K_0$, $c_1$, and $c_2$) to be determined. Then the new frequency estimate is used, and the process is continued until the wave number and frequency at the saddle point converge to within a specified tolerance. This method has also been used by Monkewitz and Sohn.[17,18]

If the imaginary part of the converged absolute frequency is greater than zero for a prescribed wake deficit $\Lambda$, the flow is absolutely unstable. To find the required wake deficit that defines the transition from a convective to an absolute instability, a smaller $\Lambda$ is used. On the other hand, if the imaginary part of the converged absolute frequency is less than zero for that wake deficit, the flow is convectively unstable. Therefore, the wake deficit must be increased. The process continues until the imaginary part of the absolute frequency is less than $10^{-6}$. By using this method, the required wake-deficit in the velocity profile to transition from a convective to an absolute instability can be determined. The absolute frequency and the required wake deficit for the chosen operating condition are $\omega_0 =$ (10.2306, 0.0) and $\Lambda_{req} = 0.790929$, respectively. Finally, for a higher wake-deficit value, the loci of the complex wave numbers are traced again. This is to check that the pinching point corresponds to an absolute instability phenomenon. Only a pinching that comes from the coalescence of one $K^+$ and one $K^-$, that originated from the upper and lower half of the $K$ plane, represents an absolute instability.[15]

## Numerical Results

Two sets of operating conditions have been considered. The first set of conditions assumes that the total temperature of the high-speed stream is constant. Under this assumption, the corresponding velocity and temperature ratios for the operating condition specified in the previous section are $\lambda_u = 0.589$ and $\lambda_T = 2.174$.

The following numerical results provide some parametric studies. The effects of the high-speed stream Mach number $M_1$ and wake deficit on the absolute frequency are first evaluated by keeping the velocity ratio ($\lambda_u = 0.589$) and temperature ratio ($\lambda_T = 2.174$) constant. The high-speed stream Mach numbers are chosen to be 2.5 and 3.5. The calculated results are plotted in Figs. 3 and 4. $u_{min}$ is the corresponding, algebraically minimum value of the streamwise velocity profile in Eq. (3). A negative value of $u_{min}$ implies reversed flow. In contrast, a positive value of $u_{min}$ indicates that the streamwise velocity profile is always positive. The continuous lines in Figs. 3 and 4 do not represent calculations at intermediate values of $M_1$, but are intended in these and in subsequent figures for qualitative guidance only. From these Figs. 3 and 4, it is clear that the absolute frequency and the required reversed flow both decrease as $M_1$ increases. For these flow conditions, reversed flow is required to induce a transition between convective and absolute instability. However, as $M_1$ increases, less reversed flow is required. This trend is consistent with the work of Peroomian and Kelly[9] for a compressible confined mixing layer with a monotonic shear-layer profile. Figures 3 and 4 indicate a nearly linear relation with $M_1$ under the constraint of a constant high-speed stream total temperature, and velocity and temperature ratios.

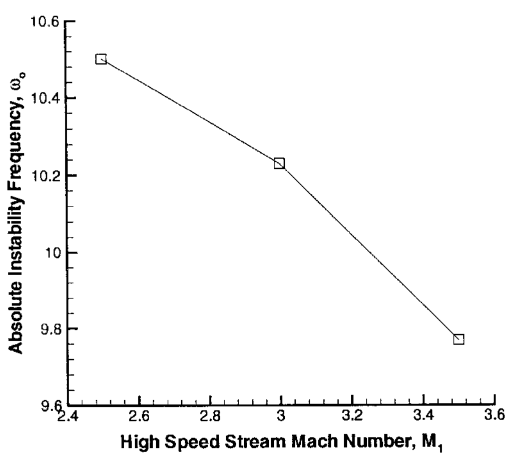


**Fig. 3 Variation with high-speed stream Mach number of absolute frequency at the onset of absolute instability, $M_1$. $\lambda_u = 0.589$, $\lambda_T = 2.174$.**

The effects of the velocity ratio $\lambda_u$ on the absolute frequency and the required wake deficit have also been investigated. In this simulation, both $M_1$ and $\lambda_T$ are held at constant values of 3.0 and 2.174, respectively. Two velocity ratios of 0.5 and 0.7 are chosen. The absolute frequency increases with increasing velocity ratio (Fig. 5). Again, reversed flow is required to change the instability type for these operating conditions. As the velocity ratio increases, more reversed flow is needed.

A third parametric study uses the temperature ratio $\lambda_T$ as a control variable. In this case, $M_1$ and $\lambda_u$ are held constant with values 3.0 and 0.589, respectively. Temperature ratios of 1.1, 1.5, and 2.174 have been considered. Figure 6 shows that an increase in the temperature ratio results in an increase in the absolute frequency. Notice that the temperature ratio can change the absolute frequency quite significantly as compared to the influence of the high-speed stream Mach number and the velocity ratio. In addition, a colder low-speed stream requires less and even no reversed flow to alter the instability type. When the temperature ratio is less than 1.58, no reversed flow is required to induce an absolute instability as shown in Fig. 7. This suggests that an adjustment of the temperature ratio is an effective way to manipulate the absolute frequency. Two examples of experiments that have used temperature as a control parameter to induce an absolute instability in a jet can be found in Sreenivasan et al.[19] and Monkewitz et al.[20]

A second set of operating conditions have been studied in which the total temperature of the high-speed stream is again assumed to be constant. In this case, the low-speed stream Mach number is fixed at 1.2. The high-speed stream Mach numbers are chosen to be greater than 2.75. The selected op-

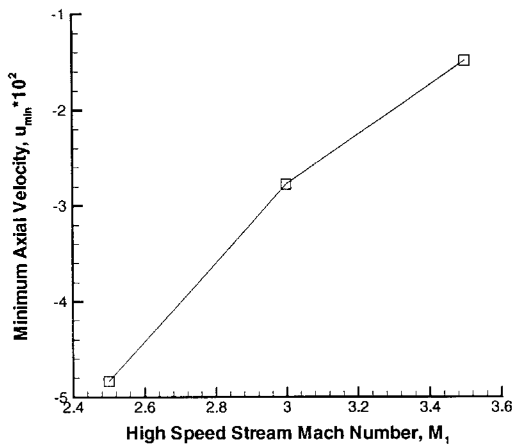


**Fig. 4 Variation with high-speed stream Mach number of minimum axial velocity required for the onset of absolute instability, $M_1$. $\lambda_u = 0.589$, $\lambda_T = 2.174$.**

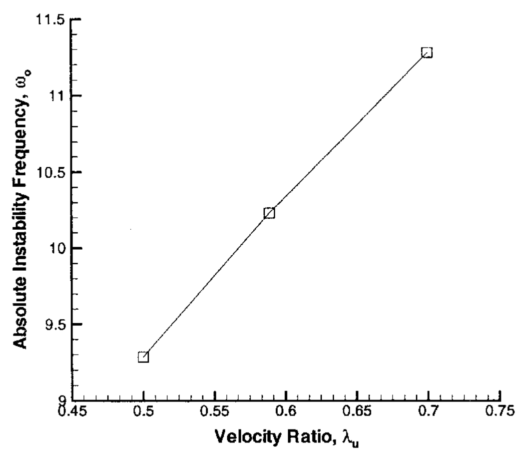


**Fig. 5 Variation with velocity ratio $\lambda_u$ of absolute frequency at the onset of absolute instability. $M_1 = 3.0$, $\lambda_T = 2.174$.**

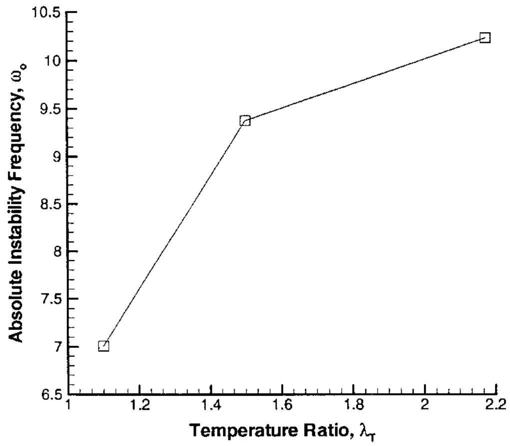


**Fig. 6 Variation with temperature ratio $\lambda_T$ of absolute frequency at the onset of absolute instability. $M_1$ = 3.0, $\lambda_u$ = 0.589.**

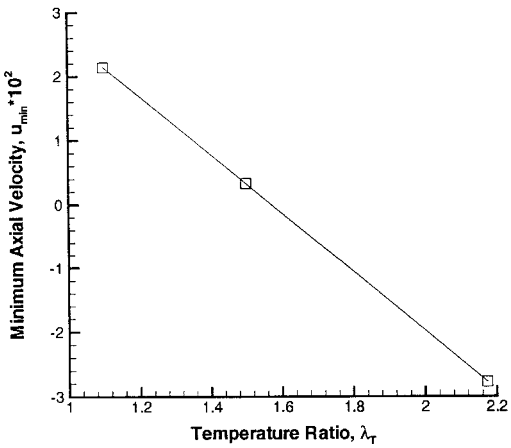


**Fig. 7 Variation with temperature ratio $\lambda_T$ of minimum axial velocity required for the onset of absolute instability. $M_1$ = 3.0, $\lambda_u$ = 0.589.**

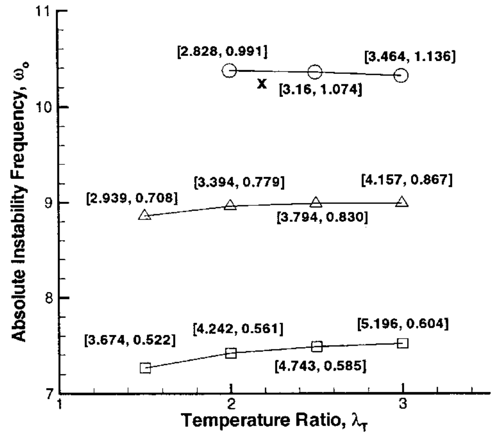


**Fig. 8 Variation of absolute frequency with temperature ratio $\lambda_T$ at the onset of absolute instability, for different velocity ratios. $T_{t1}$ is constant and $M_2$ = 1.2. -□-, $\lambda_u$ = 0.4; -△-, $\lambda_u$ = 0.5; -○-, $\lambda$ = 0.7.**

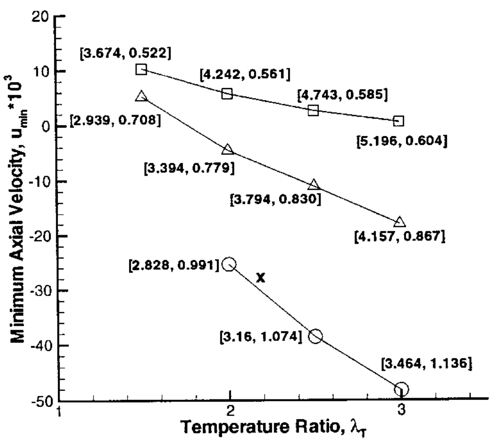


**Fig. 9 Variation with temperature ratio $\lambda_T$ of minimum axial velocity required for the onset of absolute instability, for different velocity ratios. $T_{t1}$ is constant and $M_2$ = 1.2. -□-, $\lambda_u$ = 0.4; -△-, $\lambda_u$ = 0.5; -○-, $\lambda$ = 0.7.**

erating conditions cover temperature ratios in the range from 1.5 to 3.0, velocity ratios from 0.4 to 0.6, and total temperature ratios from 0.5222 to 1.1365. This provides a good picture of how the absolute frequency varies as a function of the different operating parameters.

The absolute frequencies with zero imaginary part $\omega_0$, and the minimum velocity $u_{min}$ corresponding to the required reversed flow $\Lambda_{req}$ for the selected operating conditions are plotted in Figs. 8 and 9. For easy reference and discussion, the high-speed stream Mach number $M_1$ and the total temperature ratio between the two streams $T_{t2}/T_{t1}$ for each operating condition are enclosed in a square bracket as [$M_1$, $T_{t2}/T_{t1}$] in Figs. 8 and 9. From Fig. 8, it is clear that, for the same temperature ratio, an increase in the velocity ratio results in an increase in the absolute frequency. However, the absolute frequencies along the lines of constant velocity ratio vary slowly as the temperature ratio changes. The minimum wake-deficit velocities for these operating conditions are plotted in Fig. 9. When the velocity ratio is fixed, $u_{min}$ decreases with decreasing temperature ratios. This implies that cooling the low-speed stream facilitates the occurrence of the absolute instability. A similar result for low-speed flows was reported by Yu and Monkewitz.[21] On the other hand, when the temperature ratio is fixed, an increase in the velocity ratio results in an increase of $u_{min}$. The points in the upper left-hand corner that have low velocity and temperature ratios do not need any reversed flow to transition the type of flow instability. However, the points located in the lower right-hand corner, which have higher velocity and temperature ratios, require large amounts of reversed flow to alter the flow instabilities. In summary, increasing the velocity difference between the two streams and cooling the low-speed stream makes the transition of instabilities possible without any reversed flow. A careful review of the data in Fig. 8 reveals that the absolute frequency depends strongly on the total temperature ratio, $T_{t2}/T_{t1}$. As $T_{t2}/T_{t1}$ increases, the absolute frequency increases. However, along each line of constant velocity ratio, $T_{t2}/T_{t1}$ increases only slightly as the temperature ratio increases from 1.5 to 3.0. As pointed out previously, the absolute frequency increases slowly along the line of constant velocity ratios. Hence, the absolute frequency may be maintained constant as long as $T_{t2}/T_{t1}$ is constant. This feature accommodates a wide variation in the high-speed stream Mach number, while keeping the absolute frequency constant. Also notice that when $T_{t2}/T_{t1}$ is greater than unity, the absolute frequency decreases slightly as the temperature ratio increases. This trend is different from that when $T_{t2}/T_{t1}$ is less than unity.

To ensure that all the calculations are consistent with those from the first set of operating conditions, the absolute frequency and $u_{min}$ for the first simulation condition, $M_1$ = 3.0, $M_2$ = 1.2, $\lambda_u$ = 0.589, $\lambda_T$ = 2.174, $T_{t2}/T_{t1}$ = 1.0, are marked by crosses in Figs. 8 and 9. The values are consistent with all of the trends from the other operating conditions.

The required nondimensionalized reversed flow for all of the operating conditions is relatively small. A thick splitter plate, such as that used by Amatucci,[10] can generate a strong reversed flow as high as 23% of the high-speed stream velocity. This is definitely high enough to induce the shear-layer instability to be absolutely unstable. Hence, a thick splitter plate can serve effectively to induce an absolute instability.

## Conclusions

The relation between the absolute instability frequency and the wake deficit behind a thick splitter plate in a supersonic shear layer has been investigated. To correlate the absolute frequency with the wake deficit, two parametric studies have been conducted based on different assumptions. The first parametric study assumes a constant high-speed stream total temperature. Then one of three control variables, the high-speed stream Mach number, velocity ratio, or temperature ratio, has been varied, while the other variables have been held constant. The absolute frequency is observed to increase as the velocity ratio or temperature ratio increases. However, the absolute frequency decreases as the high-speed stream Mach number increases. For most operating conditions, a reversed flow is required to change the flow instability from convective to absolute; however, when the temperature ratio is lower than 1.58, no reversed flow is required to induce this change for the selected operating conditions.

The second set of operating conditions assumes that the total temperature of the high-speed stream is constant and that the low-speed stream Mach number is fixed at a value of 1.2. Under these assumptions, increasing the velocity difference and cooling the low-speed stream facilitates the transition of the supersonic shear layer instability from convective to absolute.

All of these numerical results indicate that the presence of the wake behind a thick splitter plate can induce an absolute instability. That the frequency of the resulting self-excitation may be manipulated suggests that this could be used to excite an unstable convective instability further downstream. The resulting potential mixing modification, caused by the excited convective instability, should be a useful direction for further investigation. The use of a thick splitter plate removes the concerns (robustness, weight, space, etc.) associated with an active device, such as a vacuum pump, without any significant performance penalty. The results of this approach to supersonic shear-layer instability are important in the design of scramjet operating conditions and geometries. The guidelines established by the present calculations provide a new direction to overcome the difficulty of supersonic mixing.

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